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COSMOLOGICAL BARYON NUMBER AND KAON CP VIOLATION FROM A COMMON SOURCE.

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Abstract

We extend an earlier model for radiatively generated fermion masses based on the Pati-Salam group to include CP violation. Spontaneous CP violation in the early universe gives rise to a complex mass matrix for heavy sterile neutrinos. The out-of-equilibrium decay of these neutrinos generates a $B - L$ asymmetry. The sterile neutrinos also act as a mass seed in generating one-loop (complex) mass matrices for the quarks. Thus, the two low energy manifestations of CP violation – the CKM phase and the baryon number asymmetry – can both be traced in a calculable way to a common source.

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1 Introduction.

All of the information we have about CP violation can be summarized by two seemingly unrelated numbers: $\epsilon = 2.26 \pm 0.02 \times 10^{-3}$ and $\eta_B \equiv n_B/n_\gamma = (2.8 - 4.0) \times 10^{-10}$ [1]. These two numbers are a measure of two seemingly unrelated phenomena: the strange decays of the K mesons and the baryon asymmetry of the universe. It is generally agreed upon that the best candidate for the source of ϵ is an unremovable phase in the quark mixing (CKM) matrix [2]; however, there is no such agreement on a single most likely source of η_B .

Although there are a number of viable models of baryogenesis most of them do not say anything about the CKM phase, a notable exception to this being some of the papers that propose that the baryon asymmetry is generated at the electroweak phase transition with the only source of CP violation being in the quark mass matrix [3]. Although electroweak baryogenesis remains an exciting possibility [4] the general consensus is that additional sources of CP violation are needed to make it work [5].

An interesting mechanism of generating the baryon asymmetry is to first generate a lepton asymmetry by the out-of-equilibrium decay of a heavy sterile neutrino and then use the $B+L$ anomaly in the Standard Model to cycle this into a baryon asymmetry [6, 7]. This scenario fits naturally into a model for radiatively generated fermion masses based on the Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ [8] that we have presented [9]. The motivation for this model was firstly to treat the quarks and leptons on a symmetric footing, and then to show that one can generate realistic masses and mixings for the Standard Model fermions without a large hierarchy of Yukawa couplings.

In this model the Standard Model fermions (including right-handed partners for the neutrinos) are supplemented by three generations of heavy sterile neutrinos. As a result of a particularly simple choice of scalar representations it is impossible for the Standard Model fermions to get masses at tree-level, and the sterile neutrinos act as mass seeds in generating finite one-loop masses for them. If we allow the sterile neutrinos to have a complex mass matrix (as the result of a spontaneous violation of CP in the early universe), then not only does their out-of-equilibrium decay generate a baryon number asymmetry, but their exchange in the one-loop diagrams also generates a complex mass matrix for the quarks. Thus one is able to derive both ϵ and η_B from the same source: a complex phase in the sterile neutrino mass matrix that gets communicated to the quark sector at the scale where lepton number breaks down. It is the purpose of this paper to explore this possibility.

In a sense this model is complementary to the models of electroweak baryo-

genesis where one starts with the CKM phase and tries to use it to generate a fermion number asymmetry. Here we start with a fermion number violating operator and try to use it to generate a phase in the CKM matrix.

In Sec. 2 we briefly review the model, its representation content and pattern of symmetry breakdown. In Sec. 3 we calculate the baryon asymmetry generated in this model in two alternative scenarios: one in which there was a period of inflation in the early universe, and one in which there wasn't. In Sec. 4 we discuss the fermion masses and CKM matrix, and illustrate with a numerical example. We conclude in Sec. 5. Some details of the scalar potential and explicit formulas for the fermion masses are presented in Appendices A and B.

2 The Model

The gauge group is $SU(4) \times SU(2)_L \times SU(2)_R$ with gauge couplings g_S , g_L and g_R . The Standard Model fermions transform in the usual representations:

$$\Psi_L^i \sim (\underline{4}, \underline{2}, \underline{1})^i \equiv \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_L^i \quad (1)$$

$$\Psi_R^i \sim (\underline{4}, \underline{1}, \underline{2})^i \equiv \begin{pmatrix} u_1 & u_2 & u_3 & N \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_R^i \quad (2)$$

where $i = 1, 2, 3$ is a generation index, and we have included a right handed neutrino N . We add to this three generations of (right-handed) sterile neutrinos

$$s^i \sim (\underline{1}, \underline{1}, \underline{1})^i. \quad (3)$$

We choose to make the matter spectrum supersymmetric,¹ so the scalars in the model transform as

$$L^i \sim (\underline{4}, \underline{2}, \underline{1})^i \equiv \begin{pmatrix} L_{u1} & L_{u2} & L_{u3} & L_\nu \\ L_{d1} & L_{d2} & L_{d3} & L_e \end{pmatrix}^i, \quad (4)$$

$$R^i \sim (\underline{4}, \underline{1}, \underline{2})^i \equiv \begin{pmatrix} R_{u1} & R_{u2} & R_{u3} & R_N \\ R_{d1} & R_{d2} & R_{d3} & R_e \end{pmatrix}^i, \quad (5)$$

and

$$\sigma^i \sim (\underline{1}, \underline{1}, \underline{1})^i. \quad (6)$$

¹This differs from the scalar spectrum in Ref. [9] where we had only two generations of L and R , and no σ . For other related models see Refs. [10, 11].

We will impose a discrete Z_3 symmetry on the gauge singlets (broken by the interactions of the Standard Model particles) under which $s^j \rightarrow e^{-i(j\pi)/3}s^j$ and $\sigma^j \rightarrow e^{i(2j\pi)/3}\sigma^j$. This permits us to make the Lagrangian CP invariant, with the vacuum expectation values of the σ^j breaking CP spontaneously as in Ref. [12].

The Yukawa interactions will then be

$$\mathcal{L}_Y = -y_i(\bar{s}^c)^i s^i \sigma^i - (\kappa_L^a)_{ij} \bar{\Psi}_L^i s^j L^a - (\kappa_R^a)_{ij}^T \bar{\Psi}_R^i (s^c)^j R^a + \text{h.c.}, \quad (7)$$

with all of the coupling constants real. We discuss the details of the scalar potential in Appendix A, noting only that we can choose parameters such that it is minimized when

$$\langle \sigma \rangle_j = \frac{v_0}{\sqrt{2}} e^{i\alpha_j}; \quad \langle R_N \rangle_j = \frac{v_R}{\sqrt{2}} \delta_{1j}; \quad \langle L_\nu \rangle_j = \frac{v_L}{\sqrt{2}} \delta_{1j} \quad (8)$$

with $|v_0| > |v_R| \gg |v_L|$. None of the Standard Model fermions get masses at tree level. Their masses are generated at one-loop by diagrams involving the sterile neutrinos (s_i) on internal lines. The pattern of symmetry breaking induced by the scalar vacuum expectation values is

$$\begin{aligned} SU(4)_C \times SU(2)_L \times SU(2)_R \times CP &\xrightarrow{v_0} SU(4)_C \times SU(2)_L \times SU(2)_R \\ &\xrightarrow{v_R} SU(3)_C \times SU(2)_L \times U(1)_Y \\ &\xrightarrow{v_L} SU(3)_C \times U(1)_Q. \end{aligned} \quad (9)$$

If we use as inputs at the electroweak scale $\alpha^{-1} = 128.5$, $s_W^2 = 0.23$, and $\alpha_S^{-1} = 8.33$, and the boundary condition $g_L(v_R) = g_R(v_R)$, then using the one-loop β functions with only gauge boson and fermion contributions, gives us $v_R = 10^{14}$ GeV. The masses of the sterile neutrinos are required to be close to v_R in order to optimize the radiative mass generation. Thus we choose $v_0 = 10^{15}$ GeV. All of the scalars have a mass $\sim v_R$ except for L_ν^1 and L_e^1 which remain light and form the components of the Standard Model Higgs boson.

3 Baryogenesis

In this section we work in the symmetric phase of the Standard Model, *i.e.*, $v_L = 0$. We shall denote the light scalar doublet with components (L_ν^1, L_e^1) as Φ^* , the (complex conjugated) Standard Model Higgs boson. The out-of-equilibrium decay that we are then concerned with is that of the lightest (right-handed) $SU(2) \times U(1)$ singlet neutrino n_1 into a left-handed lepton, l_L^i , and the Higgs boson, Φ [6]. In order to calculate this, we first need to re-express the Yukawa interactions of Fig. 1(a) involving the singlet neutrinos s_i ($i = 1, 2, 3$) in terms

of the physical (Majorana) neutrino states n_j ($j = 1, \dots, 6$) obtained when one diagonalizes the neutrino mass matrix [Fig. 1(b)].

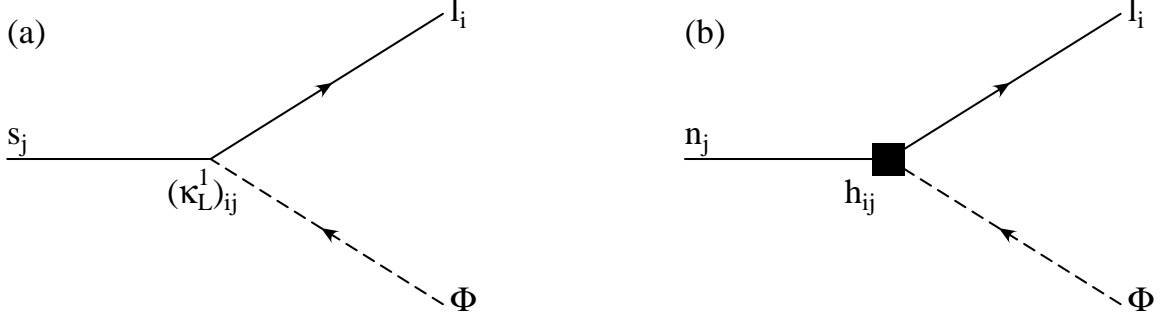


Fig. 1 Yukawa couplings of the heavy singlet neutrinos to a light left-handed lepton doublet and the Standard Model Higgs. The arrows indicate flow of left-handed isospin; $(\kappa_L^1)_{ij}$ and h_{ij} are coupling constants. (a) Coupling of the s_j . (b) Coupling of the physical states n_j .

The vacuum expectation values of Eq. (8) along with the Yukawa couplings of Eq. (7) lead to the following 6×6 mass matrix for the right-handed neutrinos in the basis (N_i, s_i) with $i = 1, 2, 3$:

$$M = \left(\begin{array}{c|ccc} 0 & & & & \frac{(\kappa_R^1)v_R}{2\sqrt{2}} \\ \hline \frac{(\kappa_R^1)^T v_R}{2\sqrt{2}} & m_{01}e^{i\alpha_1} & 0 & 0 & 0 \\ & 0 & m_{02}e^{i\alpha_2} & 0 & 0 \\ & 0 & 0 & m_{03}e^{i\alpha_3} & 0 \end{array} \right) \quad (10)$$

where $m_{0i} = y_i v_0 / \sqrt{2}$. The matrix M will be diagonalized by a 6×6 unitary matrix U with $M = U M_D U^T$ where M_D is diagonal, real and positive.

We can now obtain the Yukawa couplings of the physical heavy neutrinos, n^j , with the light left-handed leptons, l^i , and the Higgs boson Φ :

$$\mathcal{L}'_Y = -h_{ij} \bar{l}_L^i n_R^j \Phi + \text{h.c.} \quad (11)$$

where

$$h_{ij} = \sum_{k=1}^3 (\kappa_L^1)_{ik} U_{k+3,j}^* \quad (12)$$

is now a complex 3×6 matrix, and n_j are the 6 physical right-handed neutrinos arranged so that $m_1 < m_2 < \dots < m_6$.

From here the calculation proceeds just as the cases presented in the literature (except for the fact that we have to account for 6 heavy neutrinos rather than 3),

depending on whether we assume a period of inflation in the early universe [13] or not [6, 14], and we now illustrate with a numerical example that the model can naturally generate a baryon asymmetry of the correct size.

We choose as inputs the following matrices of Yukawa couplings (we will motivate this choice in the next section):

$$\kappa_L^1 = \begin{pmatrix} 0.04 & 0.03 & 0.06 \\ 0.06 & 0.42 & 0.24 \\ 0.06 & 0.08 & 3.5 \end{pmatrix}; \quad \kappa_R^1 = \begin{pmatrix} -0.06 & 0.03 & -0.04 \\ 0.06 & 0.20 & 0.16 \\ 0.06 & 0.08 & 3.5 \end{pmatrix} \quad (13)$$

$$y_1 = \sqrt{2} \times 0.025; \quad y_2 = \sqrt{2} \times 0.05; \quad y_3 = \sqrt{2} \times 0.35. \quad (14)$$

We make a simple choice for the phases α_i of Eq. (8):

$$-\pi < \alpha_1 < \pi; \quad \alpha_2 = \alpha_3 = 0. \quad (15)$$

Since n_1 is a Majorana fermion it can decay via the Yukawa interaction of Eq. (11) into both leptons and anti-leptons, *i.e.*,

$$\begin{aligned} n_1 &\rightarrow l_L^i + \Phi \\ &\rightarrow (l_L^c)^i + \Phi^*, \end{aligned} \quad (16)$$

thus violating lepton number in its decays. The tree level width of n_1 is then

$$\Gamma_t = \frac{(h^\dagger h)_{11} m_1}{8\pi} \quad (17)$$

where h is defined in Eq. (12) and

$$(h^\dagger h)_{11} = 3 \times 10^{-5}, \quad (18)$$

and

$$m_1 = 4 \times 10^{11} \text{ GeV} \quad (19)$$

for the inputs we have chosen.

Further, if CP is violated there will be an asymmetry in the decay rates into leptons and anti-leptons. Let us define an asymmetry

$$\delta = \frac{\Gamma - \Gamma^{CP}}{\Gamma + \Gamma^{CP}} \quad (20)$$

where Γ is the decay rate into leptons, and Γ^{CP} into anti-leptons. Interference between the diagrams of Figs. 2(a) and 2(b) give rise to δ which is calculated to be [6, 14]

$$\delta = \frac{1}{2\pi(h^\dagger h)_{11}} \sum_{j=1}^6 \text{Im}[(h^\dagger h)_{1j}]^2 f(m_j^2/m_1^2), \quad (21)$$

where

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] \quad (22)$$

is proportional to the imaginary part of the loop amplitude of Fig. 2(b), and arises when the l_k and Φ in the loop go on-shell.

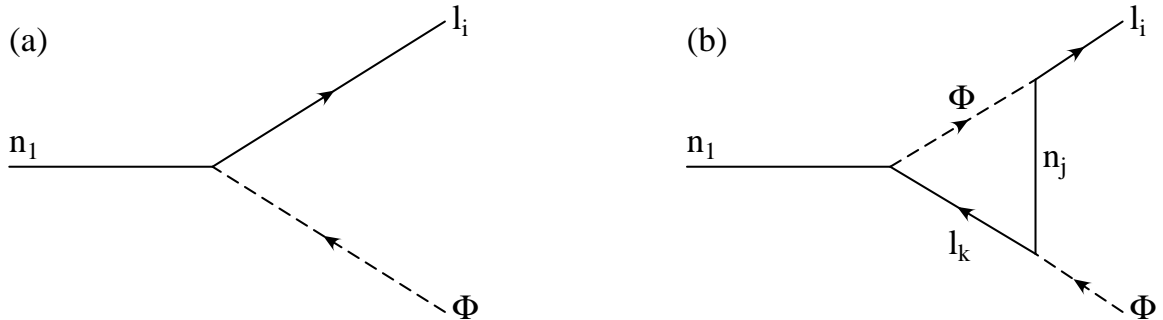


Fig. 2. Decay of the lightest physical singlet neutrino n_1 into a left-handed lepton and the Standard Model Higgs. (a) Tree level. (b) One-loop, with on-shell intermediate states.

The calculation now splits up depending on whether we want to incorporate the effects of inflation or not. We will first estimate the baryon asymmetry in the inflationary case (although this calculation is more speculative than the non-inflationary one). We will then adapt the result to the case of no inflation.²

Quantum fluctuations of the scalar field η (the inflaton), that drives inflation, give rise to density perturbations in the early universe which in turn lead to anisotropies in the cosmic microwave background radiation. Thus one can use the COBE measurements of the quadrupole moment of the microwave background [16] to constrain the parameters of inflation. For a generic inflationary potential as defined in Ref. [13], one finds

$$\frac{\mu}{M_P} \sim 10^{-4} \quad (23)$$

where μ is the inflation scale, and $M_P = 10^{19}$ GeV is the Planck mass. Thus inflation takes place at a scale $\mu \sim 10^{15}$ GeV.

The fact that inflation occurs at or below $v_0 = 10^{15}$ GeV, the scale at which CP is spontaneously broken, solves the problem of domain walls in this model. A domain wall arises whenever a discrete symmetry is spontaneously broken, and contains an unacceptably large energy density [17]. If one existed in our Hubble

²For a text book discussion of some of the estimates we make in this section and their accuracy, see Ref. [15].

volume, it would have over-closed the universe many times over [18]. Inflation at a scale below that of the discrete symmetry breakdown has the effect of diluting the density of domain walls to the level where it is extremely unlikely to find one in our observable universe.

The mass of the inflaton is

$$m_\eta \simeq \frac{\mu^2}{M_P} \simeq 10^{11} \text{ GeV} \quad (24)$$

and the reheat temperature is

$$T_{RH} \simeq (\Gamma_\eta M_P)^{1/2} \simeq \frac{\mu^3}{M_P^2} \simeq 10^7 - 10^8 \text{ GeV} \quad (25)$$

where $\Gamma_\eta \simeq m_\eta^3/M_P^2$ is the inflaton decay rate, and T_{RH} is defined as the equilibrium temperature of the relativistic decay products of the inflaton.

Thus $m_\eta \simeq 10^{11} \text{ GeV} \gtrsim m_1$, and the inflaton is heavy enough to decay into the right-handed neutrino n_1 whose subsequent decay generates a lepton asymmetry. A simple way to estimate the baryon asymmetry in this case is [13]

$$\begin{aligned} \frac{n_L}{s} &\simeq \frac{n_\eta}{s} \delta \simeq \frac{\rho_\eta}{m_\eta s} \delta \\ &\simeq \frac{T_{RH}}{m_\eta} \delta \sim (10^{-3} - 10^{-4}) \delta. \end{aligned} \quad (26)$$

Here we have used the approximation that all of inflaton's energy density, ρ_η , is instantaneously converted into the energy density of relativistic particles: $\rho_\eta \simeq g^* T_{RH}^4$. The subsequent entropy density is $s \simeq g^* T_{RH}^3$ (g^* is the number of relativistic degrees of freedom), and m_η and T_{RH} are obtained from Eq. (24) and Eq. (25) above. The baryon asymmetry is then

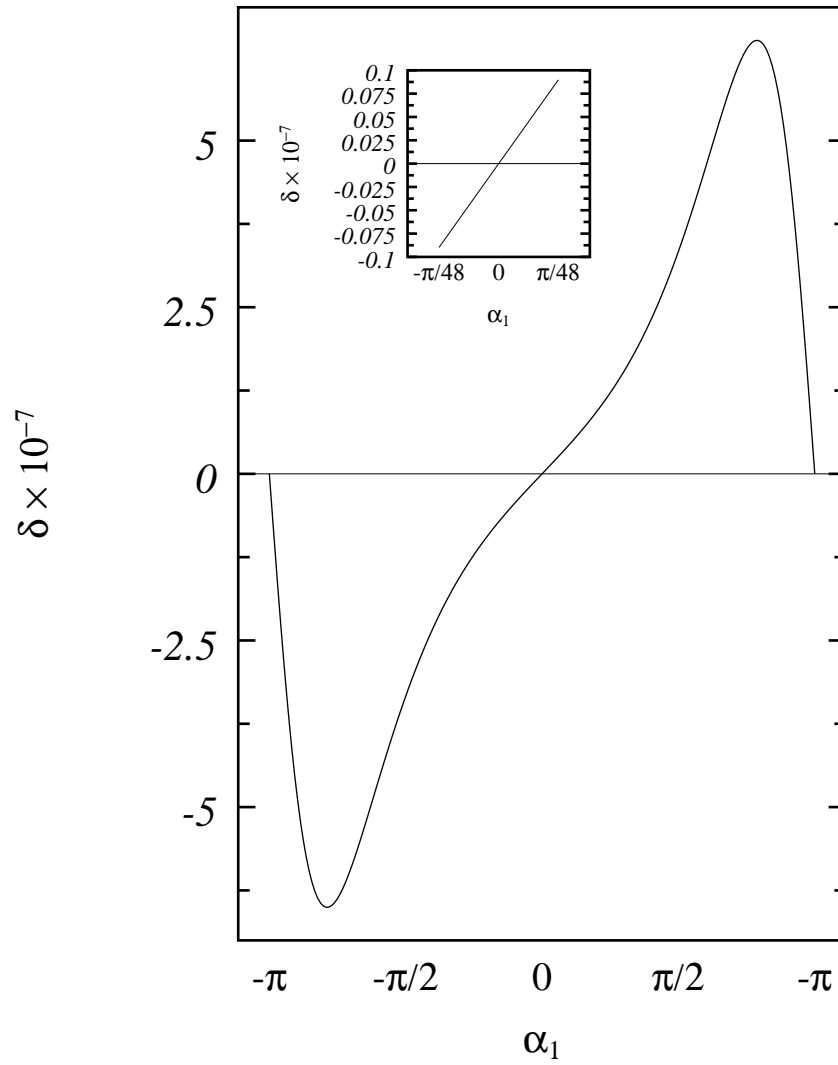
$$\eta_B \equiv \frac{n_B}{n_\gamma} = -\frac{28}{79} \times 7 \frac{n_L}{s} \sim -(10^{-3} - 10^{-4}) \delta \quad (27)$$

where we have used

$$\frac{n_B}{s} = \frac{28}{79} \frac{n_{(B-L)}}{s} \quad (28)$$

as a result of the electroweak $B + L$ violation [19], and the fact that $s = 7n_\gamma$ today. In Fig. 3 we plot the CP asymmetry δ as a function of the phase α_1 for the choice of inputs of Eqs. (13, 14). We see that the choice $\alpha_1 = -\pi/2$ gives us $\delta = -3 \times 10^{-7}$ and subsequently $\eta_B \simeq 10^{-10}$ in agreement with the observed value. We pick the value with $|\alpha_1| = \pi/2$ as it might be consistent with the notion of a “maximal” CP violation in the early universe.

Fig. 3. The CP asymmetry δ vs. the phase α_1 .



The last thing we need to ensure is that the lepton asymmetry is not washed out by the lepton number violating process $l_L l_L \rightarrow \Phi\Phi$ of Fig. 4.

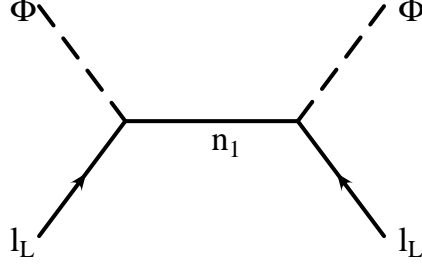


Fig. 4. Induced $\Delta L = 2$ interaction. The arrows indicate flow of lepton number.

This requires that the process be out of equilibrium when the electroweak $B + L$ violation is in equilibrium [7, 13, 19], which happens at a temperature $\tilde{T}_R \sim \alpha^2 T_{RH} \sim 10^5$ GeV, where α is a typical gauge coupling [20]. The bound one needs to satisfy is

$$\frac{m_1}{(h^\dagger h)_{11}} \geq 1.4 \times 10^{-2} \sqrt{\tilde{T}_R M_P} \quad (29)$$

which is easily satisfied for the parameters we use.

Although we prefer the inflationary scenario because it cures the domain wall problem as mentioned above, and because of its other attractive features, we now show how to adapt the calculation to the case of no inflation.

In the non-inflationary case, we first need to check if n_1 is out of equilibrium when it decays *i.e.*

$$\Gamma_t \leq H(T = m_1) \quad (30)$$

where H is the Hubble constant. This translates to

$$\frac{(h^\dagger h)_{11} m_1}{8\pi} \lesssim \frac{20 m_1^2}{M_P} \quad (31)$$

which is satisfied for our choice of inputs. The lepton asymmetry is then given by

$$\begin{aligned} \frac{n_L}{s} &\simeq \frac{n_\gamma}{s} \delta \simeq \frac{n_\gamma}{g_* n_\gamma} \\ &\simeq 10^{-2} \delta \end{aligned} \quad (32)$$

where we have used $g^* \simeq 100$ between the scales v_L and v_R . Thus we see that in this case a sufficient baryon asymmetry can be achieved with a smaller CP violation than in the inflationary case. This is borne out by Fig. 3 where we see that smaller values of α_1 indeed lead to smaller values of δ . If we choose $\alpha_1 = -\pi/96$, we get $\delta = -3 \times 10^{-9}$ and $\eta_B \simeq 10^{-10}$.

Once again we need to ensure against a wash-out of this symmetry. In this case, the $B + L$ violation is in equilibrium up to a temperature of about 10^{12} GeV, and hence in the RHS of Eq. (29) we use the temperature at which the lepton number violation occurs: $T_L = m_1 = 10^{11}$ GeV. The inequality

$$\frac{m_1}{(h^\dagger h)_{11}} \geq 1.4 \times 10^{-2} \sqrt{T_L M_P} \quad (33)$$

is indeed satisfied, and once again we find that our model can generate a sufficient baryon asymmetry.

4 Fermion Masses and the CKM matrix

The calculation for the fermion masses and mixings exactly follows that in [9], except that there are now 3 generations of scalars rather than 2, and that we now allow for CP violation, hence the orthogonal matrix O in the mass formulas is replaced by the unitary matrix U , or U^* as appropriate. We relegate the details to Appendix B, restricting the discussion in this section to qualitative features, and demonstrating with a numerical example.

As mentioned earlier, none of the Standard Model fermions get masses at tree level, even after the standard electroweak group $SU(2)_L \times U(1)_Y$ is broken. However, one-loop masses are generated by the diagrams of Figs. (5). X in Fig. 5(a) is an $SU(4)$ lepto-quark gauge boson, and Z' of Figs. 5(e), 5(f) is the neutral gauge boson that couples to the linear combination of the broken diagonal generators $U(1)_{B-L}$ and $U(1)_R$ of $SU(4)$ and $SU(2)_R$ respectively, that is orthogonal to the hypercharge $U(1)_Y$. Z of Fig. 5(f) is just the Standard Model Z boson.

One can estimate the magnitude of the mass terms from Figs. 5(a) and 5(e) involving gauge boson exchange as

$$m_{G1} \simeq \frac{g_1 g_2}{(4\pi)^2} v_L v_R \frac{m_0}{m_0^2} \simeq \frac{g_1 g_2}{(4\pi)^2} v_L \quad (34)$$

where g_1 and g_2 are the gauge couplings at the two vertices. The factor of $(4\pi)^2$ in the denominator is from the loop integral, and we have replaced the sterile neutrino mass m_0 by v_R .

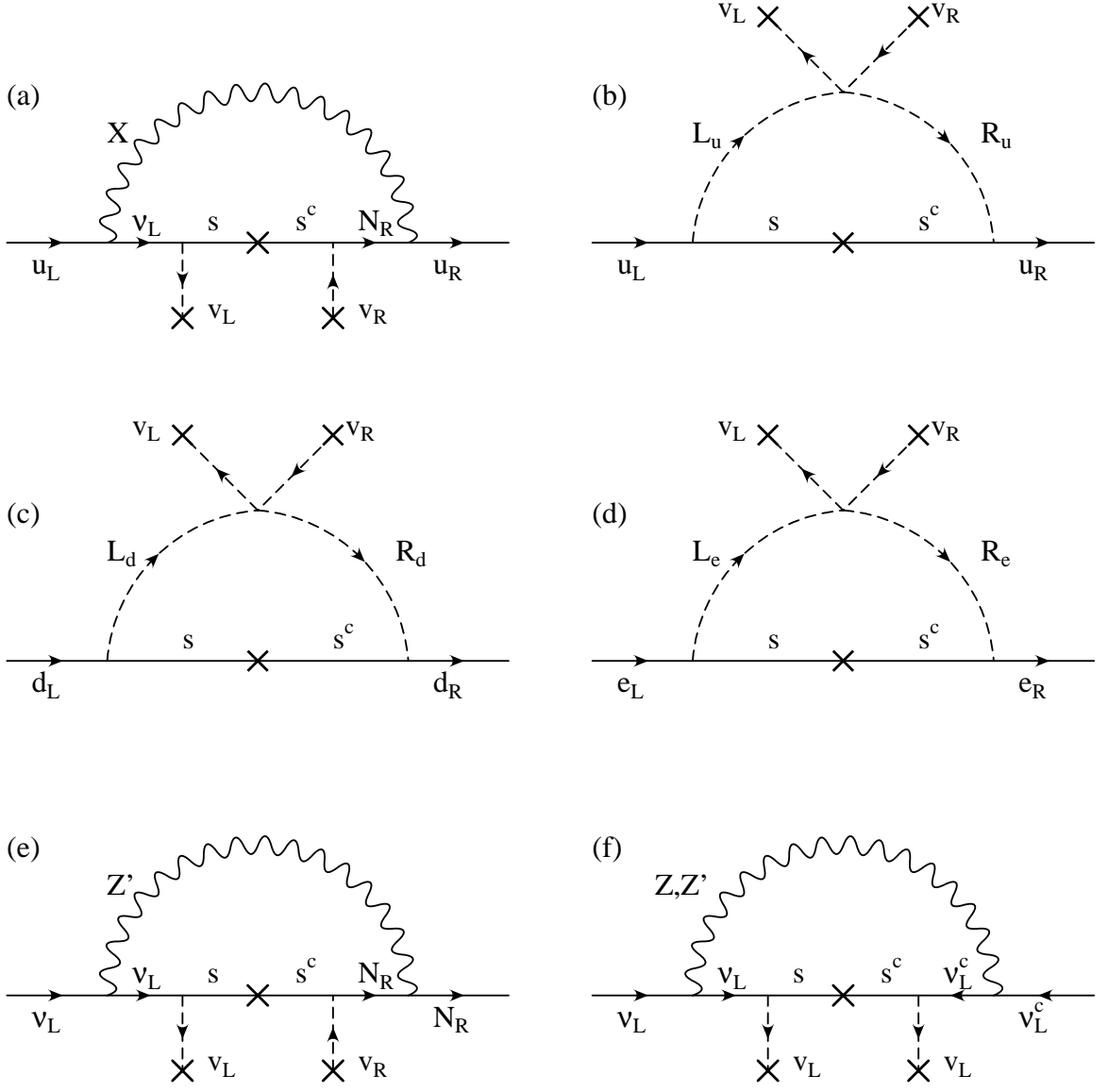


Fig. 5. One-loop processes that generate the fermion masses. The arrows here indicate the flow of left or right-handed isospin. (a) Up-type quarks with gauge bosons. (b) Up-type quarks with scalars. (c) Down-type quarks. (d) Charged leptons. (e) Dirac mass for neutrinos. (f). Majorana mass for neutrinos.

Notice that if we replace v_L on the external leg of Fig. 5(e) by the unshifted Higgs field, Φ , we get an effective interaction similar to that of Fig. 1(a), but with N_j on the external leg. One might then worry about whether it was correct to ignore this contribution to the baryogenesis calculation of the previous section. To estimate the magnitude of the effective Yukawa couplings of the N_j to the l_i and Φ , one can simply divide the result of Eq. (34) by v_L . If we use $g_1 g_2 / (4\pi) \simeq 1/40$, we get effective Yukawa couplings $y \simeq 0.002$ which are smaller than the couplings κ_L^1 of Eq. (13) by an order of magnitude. Thus their contribution to the total width of n_1 can be ignored. The fact that this process makes no contribution to the CP asymmetry can be understood by replacing all the particles in Fig. 5(e) by physical particles. Then, with an on-shell n_1 on the external leg, and n_i and Z' in the loop, the diagram can never have an imaginary part since n_1 is the lightest physical neutrino, and the particles in the loop cannot be on-shell. These estimates were confirmed by a direct calculation of the appropriate three-point form factors, and their contributions to the baryogenesis calculation.

Using arguments similar to the ones for Figs. 5(a) and 5(e), one can estimate Fig. 5(f) to give

$$m_{G2} \simeq \frac{g_1 g_2}{(4\pi)^2} \frac{v_L}{v_R} v_L, \quad (35)$$

and the scalar exchange contribution of Figs. 5(b), 5(c) and 5(d) to the fermion masses are approximately given by

$$m_S \simeq \sum_{a=1}^3 \frac{\kappa_L^a \kappa_R^a}{(4\pi)^2} v_L \quad (36)$$

One immediately notices a fundamental difference between fermion masses in this model and those in the Standard Model. Here all the fermion masses are proportional to the squares of coupling constants as opposed to depending linearly on them.³ This has the effect of reducing the hierarchy in coupling constants needed to reproduce the large hierarchy in the observed masses.

If this model were supersymmetric then the estimates for the fermion masses of the previous equations would be scaled by a factor of M_{SUSY}/v_R for $M_{SUSY} < v_R$, where M_{SUSY} is the supersymmetry breaking scale. Thus the discovery of TeV scale supersymmetry would rule out this model for generating the fermion masses.

Although all the quarks and leptons of one generation have identical Yukawa couplings in this model, the scalars to which they couple have different masses

³A more careful estimate would show that the gauge contribution of Eq. (34) is numerically similar to the scalar contribution with κ_L^1 and κ_R^1 of Eq. (36). This must be since some of the scalars L^1 and R^1 form the longitudinal components of the massive gauge bosons, *cf.* Eqs. (8,9). The choice of “gauge” or “scalar” contribution then depends on the gauge we calculate in, with the final answer being gauge independent.

and mixings determined by the parameters of the scalar potential (the explicit formulas for the masses are listed in Appendix B). Using this fact, and some of the features of Figs. 5, one can qualitatively understand how this model gives rise to the rather complicated observed spectrum of fermion masses.

The up-type quarks get masses from both gauge boson and scalar exchange [Figs. 5(a), 5(b)]. This larger number of diagrams, and the possibility of constructive or destructive interference between them allows us to generate the large hierarchy in their masses. In particular all the diagrams that contribute to the top quark mass interfere constructively to generate the large observed value, whereas destructive interference between various diagrams is responsible for making the up quark lighter than the down quark.

The down-type quarks and charged leptons both get masses only from scalar exchange, explaining the similarity in their spectra [Figs. 5(c), 5(d)].

The neutrinos get masses from gauge boson exchange [Figs. 5(e), 5(f)]. However, the large mass of the sterile neutrinos results in see-saw suppression of the physical left-handed neutrino mass to a level compatible with the MSW [21] solution to the solar neutrino problem. The neutrinos could in principle also get masses from scalar exchange, but this contribution vanishes for a natural choice of parameters in the scalar potential.

The complex phase in the CKM matrix arises as a result of the single phase α_1 in the sterile neutrino mass matrix.

Let us illustrate these features with a numerical example. We choose the following Yukawa couplings in addition to κ_L^1 , κ_R^1 , y_i of Eqs. (13, 14),

$$\begin{aligned} \kappa_L^2 &= \begin{pmatrix} 0.04 & 0.03 & 0.06 \\ 0.03 & 0.4 & 0.2 \\ 0.1 & 3.5 & 3.5 \end{pmatrix}; & \kappa_R^2 &= \begin{pmatrix} 0.04 & 0.03 & 0.06 \\ -0.03 & 0.4 & -0.2 \\ 0.1 & 3.5 & 3.5 \end{pmatrix} \\ \kappa_L^3 &= \begin{pmatrix} -0.2 & 0.02 & -0.2 \\ -0.2 & 0.4 & -0.4 \\ 3.5 & 3.5 & 3.5 \end{pmatrix}; & \kappa_R^3 &= \begin{pmatrix} 0.2 & -0.02 & 0.2 \\ -0.4 & -2.0 & -1.0 \\ 3.5 & 3.5 & 3.5 \end{pmatrix} \end{aligned} \quad (37)$$

The procedure we used to pick the Yukawa couplings was the following. The diagonal entries of $\kappa_{L,R}^{(1,2)}$ were arbitrarily chosen to reflect the hierarchy $1 : \lambda : \lambda^2$ with $\lambda = 0.1$, going from the third generation to the first, in accordance with our policy of not allowing too large a hierarchy in the couplings. The scale was set by the (3, 3) entry which was picked to be 3.5, the largest value with which we felt comfortable doing perturbation theory. The first row and column were chosen to reflect the magnitude of the (1,1) element, and the (2,3) and (3,2) elements were arbitrarily chosen to reflect either the (2,2) or the (3,3) elements. Some of the entries were randomly assigned minus signs (except the entries in the third row, which contribute to the top quark mass, and were all required to be positive). For

the $\kappa_{L,R}^3$, all the elements of the third row were set to 3.5 in order to maximize the top quark mass, and the elements of the first and second rows were varied in order to fit the data.

Given the inputs above, for a choice of parameters of the scalar potential similar to those of [9] one obtains the following fermion masses and mixings (at the electroweak scale ~ 100 GeV).

$$m_u = 7.0 \text{ MeV}; \quad m_c = 1.2 \text{ GeV}; \quad m_t = 160 \text{ GeV}. \quad (38)$$

$$m_d = 8.0 \text{ MeV}; \quad m_s = 140 \text{ MeV}; \quad m_b = 3.9 \text{ GeV}. \quad (39)$$

The absolute values of the CKM matrix elements are

$$|V| = \begin{pmatrix} 0.98 & 0.21 & 0.004 \\ 0.21 & 0.98 & 0.06 \\ 0.01 & 0.06 & 1.0 \end{pmatrix} \quad (40)$$

As a measure of the CP violation in the CKM matrix we first calculate the parameter J_{CP} , where

$$J_{CP} = \text{Im}[V_{cb}V_{us}V_{ub}^*V_{cs}^*] \quad (41)$$

is twice the area of the unitarity triangle of CKM matrix elements, and is a rephasing invariant measure of the CP violation in the CKM matrix with 3 generations of quarks. We plot J_{CP} as a function of the phase α_1 in Fig. 6. In the Wolfenstein parametrization of the CKM matrix [22], we have $J_{CP} = A^2\lambda^6\eta \simeq 4 \times 10^{-5}$ where we have used $A = 0.85$, $\lambda = 0.22$, and $\eta \simeq 0.5$. For our choice of inputs, and with $\alpha_1 = -\pi/2$ we see from Fig. 6 that

$$J_{CP} = 3 \times 10^{-5}, \quad (42)$$

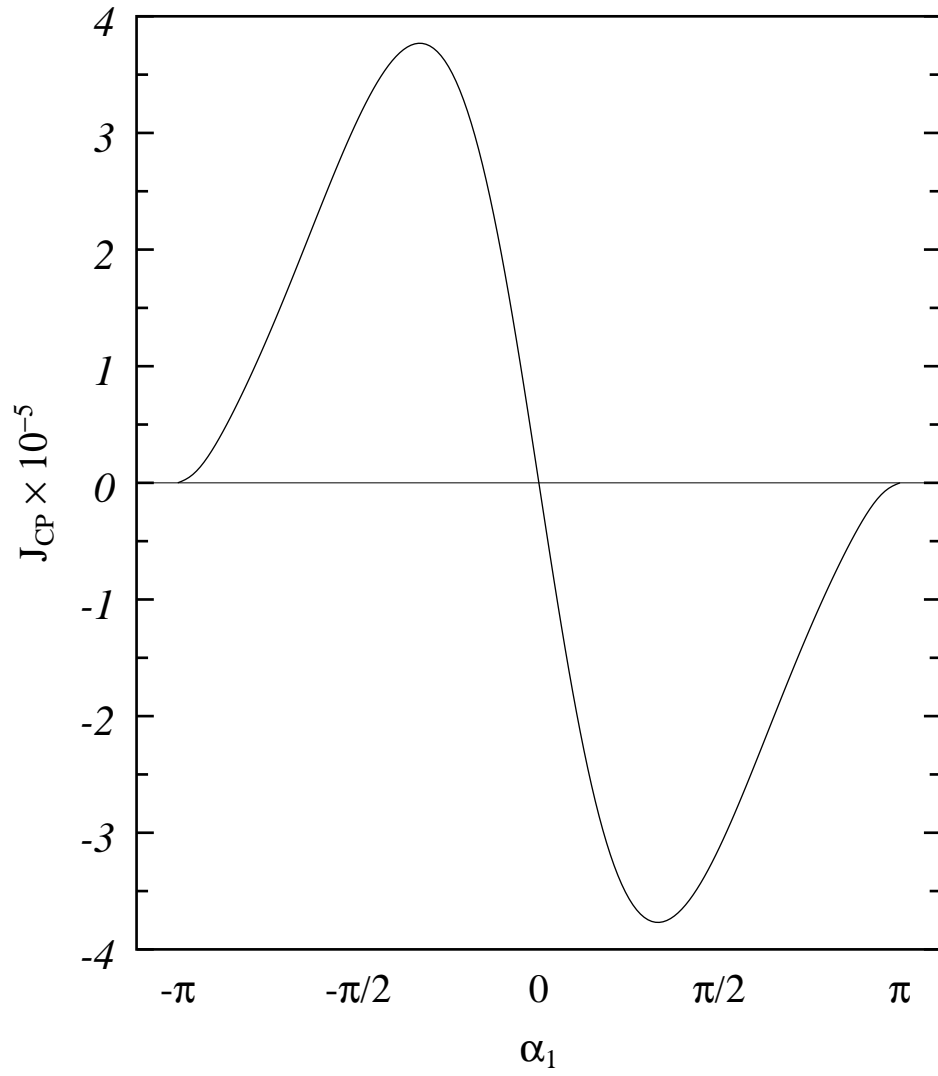
which has the correct magnitude and sign.

Notice that if we attach a photon to the charged particles on the internal lines of the diagrams in Fig. 5, we could induce an electric dipole moment for the quarks (and charged leptons) at one-loop order. This is to be contrasted with the Standard Model, where the fermion electric dipole moments arise only at three-loop order [23, 24]. An estimate of the dipole moment is then

$$d_\psi \sim \frac{m_\psi}{v_R^2} \text{ e} \cdot \text{cm} \quad (43)$$

where m_ψ is the mass of the fermion in question. This is well below the estimated Standard Model value for the quarks [25], and hence the low energy hadronic CP violation in this model is effectively of the CKM type.

Fig. 6. The CKM matrix invariant J_{CP} vs. the phase α_1 .



In order to estimate the kaon CP violating parameter ϵ , we assume that the top quark contribution dominates the box diagram that leads to $K^0 - \bar{K}^0$ mixing. In this case one has [26, 27]

$$|\epsilon| \simeq C_\epsilon B_K J_{CP} |V_{ts}|^2 \simeq 3 \times 10^{-3} \quad (44)$$

where we have used

$$C_\epsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M} = 3.8 \times 10^4, \quad (45)$$

and $B_K = 0.65$ (B_K measures the accuracy of the vacuum saturation approximation).

In the lepton sector we obtain

$$m_e = 0.7 \text{ MeV}; \quad m_\mu = 90 \text{ MeV}; \quad m_\tau = 1.3 \text{ GeV}. \quad (46)$$

$$m_{\nu_e} = 7 \times 10^{-5} \text{ eV}; \quad m_{\nu_\mu} = 4 \times 10^{-3} \text{ eV}; \quad m_{\nu_\tau} = 3 \times 10^{-2} \text{ eV}. \quad (47)$$

The absolute values of the lepton mixing matrix are

$$|V_\nu| = \begin{pmatrix} 0.92 & 0.39 & 0.03 \\ 0.39 & 0.91 & 0.12 \\ 0.03 & 0.12 & 0.99 \end{pmatrix} \quad (48)$$

We should point out that since the neutrino masses depend sensitively on the scale v_R via the see-saw mechanism, the above values should be taken as representative of the range the actual masses should lie in. This prediction of the scale of the neutrino masses is then fairly robust since they are generated only by gauge boson exchange, and hence independent of the Yukawa matrices $\kappa_{L,R}^{(2,3)}$ and details of the scalar potential (they do depend on $\kappa_{L,R}^1$ which, however, are constrained by the baryogenesis calculation of Sec. 3). It is interesting to note, then, that the ν_e and ν_μ masses lie in the correct range to explain the solar neutrino deficit as a result of the MSW [21] effect, with the mixing angle being compatible with the large angle solution [28]. We should point out, however, that the prediction for this mixing angle is not as robust as that of the masses since it is dominated by the contribution of the unitary matrix that diagonalizes the charged lepton mass matrix, and hence dependent on details of the scalar sector. Although the $\nu_\mu - \nu_\tau$ mass difference lies in the correct range for the proposed solution to the atmospheric neutrino problem, the required mixing angle is generically much smaller than that required by the theoretical fits to the data [28].

This model also predicts CP violation in the lepton sector, due to complex phases in the lepton mixing matrix. However, these effects are suppressed by the smallness of the neutrino masses, and hence, unobservable. If one applies the estimate of Eq. (43) for the electric dipole moments of the charged leptons, one gets a value larger than what one would calculate just using the phases in the mixing matrix [23], however it is still much too small to be observed.

Thus we are able to generate realistic fermion masses and mixings within our stated objective of working in a quark-lepton symmetric model without a large hierarchy of coupling constants. Not only that, but starting with the same phase $\alpha_1 = -\pi/2$ in the sterile neutrino mass matrix that generates a lepton (and hence baryon) asymmetry, we are also able to generate the correct amount of CP violation in the CKM matrix Eq. (42).

All of the inputs to the model were defined at the high energy scale $v_R = 10^{14}$ GeV. We would like to remark on the procedure we have followed in obtaining the outputs at 100 GeV. The effective theory below the scale v_R is just the Standard Model and hence we run the masses using Standard Model β functions. Effectively this results in a scaling of the light quark masses by a factor of 2.5 to account for the QCD effects, whereas the top quark mass scales by a factor of 1.7, on account of its slower running due to its large Yukawa coupling to the Standard Model Higgs. None of the mixing angles, nor the lepton masses run to the degree of accuracy we're concerned with.

As we have mentioned, the discovery of low energy supersymmetry would rule out this mechanism of mass generation. However the scenario whereby a CP violating mass matrix for sterile neutrinos is the source of both the baryon asymmetry and the CKM phase may still remain viable. We are currently studying the possibility of building this scenario into a supersymmetric model. Another possibility we are studying is the extension of this model to include family symmetries for the Standard Model fermions in order to reduce the arbitrariness of the Yukawa couplings. The fact that the Yukawa couplings in this model only range from $\mathcal{O}(1)$ to $\mathcal{O}(\lambda^2)$ with $\lambda = 0.1$, may allow a simple assignment of family charge to the Standard Model fermions, and the possible use of non-abelian family symmetries [29].

5 Conclusions

We have presented a simple extension of the Standard Model based on the Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$. CP is spontaneously broken in the early universe, and the CP violation manifests itself as a single phase of $-\pi/2$ in the mass matrix of the sterile neutrinos that are present in our model. With this

phase as the source, we can generate both the baryon asymmetry of the universe, $\eta_B \simeq 10^{-10}$, and $\epsilon \simeq 10^{-3}$, the CP violating asymmetry in kaon decays.

Masses for the fermions are generated radiatively at one-loop order, and we can obtain realistic masses and mixings without the large hierarchy of coupling constants required in the Standard Model. The neutrino masses lie in the correct range for the MSW solution to the solar neutrino problem. There will be CKM-type CP violation in the lepton sector, however the smallness of the neutrino masses makes any effects unobservable.

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Appendix A: The Scalar Potential

Given the hierarchy $v_0 > v_R \gg v_L$, the simplest way to study the scalar potential is to analyze the three sectors comprising σ^i , R^i , and L^i independently. While this suffers from a lack of generality, it makes the analysis extremely transparent without affecting our results.

For the σ^i we work with a simplified version of the Z_3 invariant scalar potential of Ref. [12].

$$\begin{aligned} V(\sigma) &= -2m_1^2(\sigma_1^\dagger\sigma_1) - 2m_2^2(\sigma_2^\dagger\sigma_2) - 2m_3^2(\sigma_3^\dagger\sigma_3) \\ &\quad + a_1(\sigma_1^\dagger\sigma_1)^2 + a_2(\sigma_2^\dagger\sigma_2)^2 + a_3(\sigma_3^\dagger\sigma_3)^2 \\ &\quad + [C_1(\sigma_1^\dagger\sigma_2)(\sigma_1^\dagger\sigma_3) + C_2(\sigma_2^\dagger\sigma_1)(\sigma_2^\dagger\sigma_3) + C_3(\sigma_3^\dagger\sigma_1)(\sigma_3^\dagger\sigma_2) + \text{h.c.}] \end{aligned} \quad (49)$$

We then assume the vacuum expectation value $\langle\sigma\rangle_j = v_j e^{i\alpha_j}/\sqrt{2}$, and study the equations to minimize the potential. In particular, for the choice $C_2 = C_3 = 0$, $m_i \simeq 10^{15}$ GeV, we recover

$$v_1^2 = v_2^2 = v_3^2 = v_0^2 = \frac{m_1^2}{a_1 - C_1}; \quad \alpha_1 = -\frac{\pi}{2}, \quad \alpha_2 = \alpha_3 = 0 \quad (50)$$

which are the values used in Eqs. (8, 15).

For the R^i and L^i , we specialize to the one-family case, since $L^1 \equiv L$ and $R^1 \equiv R$ are the only scalars that get vacuum expectation values (Eq. (8)).

$$V(R) = -2\mu_R^2 R_{i\alpha} R^{i\alpha} + \lambda_{R1} R_{i\alpha} R^{i\alpha} R_{j\beta} R^{j\beta} + \lambda_{R2} R_{i\alpha} R^{j\alpha} R_{j\beta} R^{i\beta} + \lambda_{R3} R_{i\alpha} R^{j\alpha} R_{\beta}^i R_j^\beta \quad (51)$$

$$V(L) = -2\mu_L^2 L_{i\alpha} L^{i\alpha} + \lambda_{L1} L_{i\alpha} L^{i\alpha} L_{j\beta} L^{j\beta} + \lambda_{L2} L_{i\alpha} L^{j\alpha} L_{j\beta} L^{i\beta} + \lambda_{L3} L_{i\alpha} L^{j\alpha} L_{\beta}^i L_j^\beta. \quad (52)$$

Here $i = 1, 2$ is the $SU(2)_L$ or $SU(2)_R$ index, and $\alpha = 1, 2, 3, 4$ is the $SU(4)$ index. $L^{i\alpha} = (L_{i\alpha})^*$ and $L_{\alpha}^i = \epsilon^{ij} L_{j\alpha}$, and similarly for $L \rightarrow R$. We can then generalize the arguments of Ref. [30] to show that if the following conditions are satisfied:

$$\begin{aligned} \lambda_{R2} &< 0; \quad \lambda_{R1} + \lambda_{R2} > 0; \quad \lambda_{R3} > \lambda_{R2} \quad \text{or} \quad |\lambda_{R3}| > 2\lambda_{R1} + \lambda_{R2} \\ \lambda_{L2} &< 0; \quad \lambda_{L1} + \lambda_{L2} > 0; \quad \lambda_{L3} > \lambda_{L2} \quad \text{or} \quad |\lambda_{L3}| > 2\lambda_{L1} + \lambda_{L2} \end{aligned} \quad (53)$$

the absolute minimum of the potential is at

$$\begin{aligned} \langle R \rangle &= \begin{pmatrix} 0 & 0 & 0 & v_R/\sqrt{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \langle L \rangle &= \begin{pmatrix} 0 & 0 & 0 & v_L/\sqrt{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (54)$$

with

$$\begin{aligned} (\lambda_{R1} + \lambda_{R2})v_R^2 &= 2\mu_R^2 \\ (\lambda_{L1} + \lambda_{L2})v_L^2 &= 2\mu_L^2. \end{aligned} \quad (55)$$

For $\mu_R \simeq 10^{14}$ GeV and $\mu_L \simeq 100$ GeV, we then obtain $v_R \simeq 10^{14}$ GeV and $v_L \simeq 100$ GeV. There will be other constraints on the parameters of the scalar potential to ensure $\langle L \rangle^j = \langle R \rangle^j = 0$ for $j \neq 1$, which are easily satisfied for natural choices of the parameters. The only light scalars are then L_ν^1 and L_e^1 , with the rest having masses $\sim v_R$.

We would like to point out that in order for scalar exchange to generate fermion masses one actually needs $L^i - R^i$ mixing to interchange left and right weak isospin as one can see from Figs. 5(b), 5(c), 5(d). What we do in the actual calculation is to allow individual couplings to be non-zero, but require that the combination of constants that couple v_L to v_R in the equations that determine the minimum to add up to zero, thus leaving Eq. (55) unchanged even in the more general case (see Ref. [9] for details).

Another point we would like to make is that one could in principle start with a scalar potential that was $L - R$ symmetric, and use the couplings of the L^i , R^i to the σ^i to break this symmetry as in Ref. [31].

Appendix B: Details on the Fermion Masses

In this appendix we would like to fill in some of the details, and list the explicit formulas used to calculate the fermion masses.

All the calculations were done at zero external momentum, and with physical particles propagating in the loop diagrams of Fig. (5). Thus, before we get to the formulas for calculating the Standard Model fermion masses, we need to obtain the masses and mixing angles for the singlet neutrinos, gauge bosons, and scalars that propagate in the the diagrams of Fig. (5).

The vacuum expectation values for the scalars, σ^i , L^1 , and R^1 [Eqs. (8, 50, 54)], along with the Yukawa couplings of Eq. (7) generate the following 9×9 mass matrix for the neutrinos in the basis (ν_i^c, N_i, s_i) for $i = 1, 2, 3$,

$$M = \left(\begin{array}{c|c|ccc} 0 & 0 & \frac{(\kappa_L^1)v_L}{2\sqrt{2}} & & \\ \hline 0 & 0 & \frac{(\kappa_R^1)v_R}{2\sqrt{2}} & & \\ \hline \frac{(\kappa_L^1)^T v_L}{2\sqrt{2}} & \frac{(\kappa_R^1)^T v_R}{2\sqrt{2}} & m_{01}e^{i\alpha_1} & 0 & 0 \\ & & 0 & m_{02}e^{i\alpha_2} & 0 \\ & & 0 & 0 & m_{03}e^{i\alpha_3} \end{array} \right) \quad (56)$$

where $m_{0i} = y_i v_0 / \sqrt{2}$. The matrix M will be diagonalized by a 9×9 unitary matrix U with $M = U M_D U^T$ where M_D is diagonal, real and positive. This matrix has eigenvalues $m_1 = m_2 = m_3 = 0$ corresponding to the physical states n_1, n_2, n_3 that are mostly admixtures of the ν_i , and $m_4, \dots, m_9 \simeq v_R$ for the physical states n_4, \dots, n_9 that are mostly admixtures of the N_i and s_i . The mixing between the (ν_i) and the (N_i, s_i) is of order v_L/v_R .

The gauge bosons for the gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ are:

$$\hat{G}_\mu = \frac{1}{2} \left(\begin{array}{cccc} G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}G_{12\mu}^+ & \sqrt{2}G_{13\mu}^+ & \sqrt{2}X_{1\mu}^+ \\ \sqrt{2}G_{12\mu}^- & -G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}G_{23\mu}^+ & \sqrt{2}X_{2\mu}^+ \\ \sqrt{2}G_{13\mu}^- & \sqrt{2}G_{23\mu}^- & -\frac{2G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2}X_{3\mu}^+ \\ \sqrt{2}X_{1\mu}^- & \sqrt{2}X_{2\mu}^- & \sqrt{2}X_{3\mu}^- & -\frac{3B_\mu}{\sqrt{6}} \end{array} \right), \quad (57)$$

$$\hat{W}_{L\mu} = \frac{1}{2} \left(\begin{array}{cc} W_{L\mu}^0 & \sqrt{2}W_{L\mu}^+ \\ \sqrt{2}W_{L\mu}^- & -W_{L\mu}^0 \end{array} \right), \quad (58)$$

and

$$\hat{W}_{R\mu} = \frac{1}{2} \left(\begin{array}{cc} W_{R\mu}^0 & \sqrt{2}W_{R\mu}^+ \\ \sqrt{2}W_{R\mu}^- & -W_{R\mu}^0 \end{array} \right), \quad (59)$$

where the G_μ are the gluons, B_μ is the diagonal gauge boson that couples to $B - L$, and the X_μ are the lepto-quarks.

As a result of the spontaneous symmetry breaking outlined in Eq. (9) the charged gauge bosons get the following masses:

$$M_X^2 = \frac{g_S^2}{4}[v_R^2 + v_L^2]; \quad M_{W_L}^2 = \frac{g_L^2}{4}v_L^2; \quad M_{W_R}^2 = \frac{g_R^2}{4}v_R^2. \quad (60)$$

The neutral gauge bosons have the mass squared matrix

$$M_0 = \frac{1}{8} \begin{pmatrix} g_L^2 v_L^2 & 0 & -(3/\sqrt{6})g_L g_S v_L^2 \\ 0 & g_R^2 v_R^2 & -(3/\sqrt{6})g_R g_S v_R^2 \\ -(3/\sqrt{6})g_L g_S v_L^2 & -(3/\sqrt{6})g_R g_S v_R^2 & (3/2)g_S^2(v_R^2 + v_L^2) \end{pmatrix} \quad (61)$$

in the basis (W_L^0, W_R^0, B^0) , with eigenvalues

$$M_\gamma^2 = 0; \quad M_Z^2 = \frac{v_L^2}{4}[g_L^2 + \frac{3g_R^2 g_S^2}{2g_R^2 + 3g_S^2} + \mathcal{O}(v_L^2/v_R^2)]; \quad M_{Z'}^2 = \frac{v_R^2}{8}[2g_R^2 + 3g_S^2 + \mathcal{O}(v_L^2/v_R^2)] \quad (62)$$

and eigenvectors

$$\begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} s_W & s_W & \sqrt{c_{2W}} \\ c_W & -t_W s_W & -t_W \sqrt{c_{2W}} \\ 0 & -\sqrt{c_{2W}}/c_W & t_W \end{pmatrix} \begin{pmatrix} W_L^0 \\ W_R^0 \\ B_0 \end{pmatrix}. \quad (63)$$

For $v_L \ll v_R$, the usual electroweak relation $M_W^2 = M_Z^2 c_W^2$ is still maintained where we define

$$g_L^2 = \frac{e^2}{s_W^2} \Rightarrow s_W^2 = \frac{3g_R^2 g_S^2}{3g_R^2 g_S^2 + 2g_L^2 g_R^2 + 3g_L^2 g_S^2}. \quad (64)$$

For the scalars we make the simplifying assumption that the different generations don't mix. In this case, within each generation, we will get a 2×2 mass matrix for each of the differently charged scalars. As an example of the notation we will use, consider the scalars L_u^3, R_u^3 . Diagonalizing their mass matrix will give us two physical states with masses M_{u3}, M'_{u3} and a mixing angle s_{u3} . The masses will generally be of order v_R and the mixing angles of order v_L/v_R . Care should be taken to spot the scalars that get absorbed to form the longitudinal components of the massive gauge bosons, and to treat them appropriately. Since we do our calculation in 't Hooft-Feynman gauge, these scalars are simply assigned the mass of the corresponding gauge bosons.

Having enumerated the physical neutrinos, gauge bosons, and scalars, we can finally list the formulas used to calculate the fermion masses (the indices (a, b)

in the following formulas are generation indices). For the up-type quarks we get from Fig. 5(a):

$$m^{ab} = 2 \frac{g_S^2}{(4\pi)^2} \sum_{i=1}^9 U_{a,i} U_{b+3,i} m_i \left[\frac{m_i^2}{m_i^2 - M_X^2} \ln \left(\frac{m_i^2}{M_X^2} \right) \right]. \quad (65)$$

From Fig. 5(b) we get:

$$\begin{aligned} m_j^{ab} &= \sum_{m,n=1}^3 \frac{\kappa_{Lj}^{am} \kappa_{Rj}^{bn}}{(4\pi)^2} s_{uj} \sum_{i=1}^9 U_{m+6,i}^* U_{n+6,i}^* m_i \\ &\quad \left[\frac{M_{uj}^2}{m_i^2 - M_{uj}^2} \ln \left(\frac{m_i^2}{M_{uj}^2} \right) - \frac{M'_{uj}{}^2}{m_i^2 - M'_{uj}{}^2} \ln \left(\frac{m_i^2}{M'_{uj}{}^2} \right) \right], \end{aligned} \quad (66)$$

and we sum over $j = 1, 2, 3$ for the three generations of scalars.

For the down-type quarks, Fig. 5(c) gives us

$$\begin{aligned} m_j^{ab} &= \sum_{m,n=1}^3 \frac{\kappa_{Lj}^{am} \kappa_{Rj}^{bn}}{(4\pi)^2} s_{dj} \sum_{i=1}^9 U_{m+6,i}^* U_{n+6,i}^* m_i \\ &\quad \left[\frac{M_{dj}^2}{m_i^2 - M_{dj}^2} \ln \left(\frac{m_i^2}{M_{dj}^2} \right) - \frac{M'_{dj}{}^2}{m_i^2 - M'_{dj}{}^2} \ln \left(\frac{m_i^2}{M'_{dj}{}^2} \right) \right]. \end{aligned} \quad (67)$$

For the charged leptons, from Fig. 5(d), we get

$$\begin{aligned} m_j^{ab} &= \sum_{m,n=1}^3 \frac{\kappa_{Lj}^{am} \kappa_{Rj}^{bn}}{(4\pi)^2} s_{ej} \sum_{i=1}^9 U_{m+6,i}^* U_{n+6,i}^* m_i \\ &\quad \left[\frac{M_{ej}^2}{m_i^2 - M_{ej}^2} \ln \left(\frac{m_i^2}{M_{ej}^2} \right) - \frac{M'_{ej}{}^2}{m_i^2 - M'_{ej}{}^2} \ln \left(\frac{m_i^2}{M'_{ej}{}^2} \right) \right]. \end{aligned} \quad (68)$$

Here, however, there is no contribution for $j = 1$ since the L_e^1 and R_e^1 form the longitudinal components of the W_L and W_R , respectively, and don't mix at tree level.

For the neutrinos, Fig. 5(e) gives us the following Dirac mass matrix

$$m^{ab} = \frac{g_1 g_2}{(4\pi)^2} \sum_{i=1}^9 U_{a,i} U_{b+3,i} m_i \left[\frac{m_i^2}{m_i^2 - M_Z^2} \ln \left(\frac{m_i^2}{M_Z^2} \right) \right]. \quad (69)$$

where

$$g_1 = \frac{3g_S}{\sqrt{6}}; \quad g_2 = \frac{3t_W g_S}{\sqrt{6}} + \frac{g_L \sqrt{c_{2W}}}{c_W}. \quad (70)$$

Fig. 5(f) leads to the following Majorana mass matrix for the left-handed neutrinos,

$$\begin{aligned}
m^{ab} &= 2 \frac{g_1^2}{(4\pi)^2} \sum_{i=1}^9 U_{a,i} U_{b,i} m_i \left[\frac{m_i^2}{m_i^2 - M_Z^2} \ln \left(\frac{m_i^2}{M_Z^2} \right) \right] \\
&\quad + 2 \frac{g_0^2}{(4\pi)^2} \sum_{i=1}^9 U_{a,i} U_{b,i} m_i \left[\frac{m_i^2}{m_i^2 - M_Z^2} \ln \left(\frac{m_i^2}{M_Z^2} \right) \right]
\end{aligned} \tag{71}$$

where

$$g_0 = \frac{g_L}{c_W}. \tag{72}$$

There will be a similar Majorana mass matrix for the right-handed neutrinos, N_a , with $g_1 \rightarrow g_2$, and $a, b \rightarrow a + 3, b + 3$ in the first term of Eq. (71), and no contribution from the second term. These 3×3 mass matrices for the neutrinos fill in the appropriate zero blocks in the 9×9 mass matrix of Eq. (56), which is then rediagonalized, resulting in small but finite see-saw suppressed masses for the left-handed neutrinos.

References

- [1] Particle Data Group, L. Montanet *et al.*, *Phys. Rev.* **D50**, 1174 (1994).
- [2] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Th. Phys.* **49**, 652 (1973).
- [3] G. R. Farrar and M. E. Shaposhnikov, *Phys. Rev. Lett.* **70**, 2833 (1993).
- [4] For a review see A. G. Cohen, D. B. Kaplan, and A. E. Nelson, *Ann. Rev. Nucl. Part. Sci.* **43**, 27 (1993).
- [5] P. Huet and E. Sather, *Phys. Rev.* **D51**, 379 (1995).
- [6] M. Fukugita and T. Yanagida, *Phys. Lett.* **B174**, 45 (1986).
- [7] M. Fukugita and T. Yanagida, *Phys. Rev.* **D42**, 1285 (1990).
- [8] J. C. Pati and A. Salam, *Phys. Rev.* **D10**, 275 (1974).
- [9] M. P. Worah, University of Chicago preprint EFI-95-04, hep-ph/ 9502222, February (1995); submitted to *Phys. Rev.* **D**.
- [10] K. S. Babu, B. S. Balakrishna and R. N. Mohapatra, *Phys. Lett.* **B237**, 221 (1990).

- [11] R. R. Volkas, Melbourne U. and Cambridge U. preprint DAMTP-94-106, hep-ph/9507215, July (1995).
- [12] G. C. Branco and A. I. Sanda, *Phys. Rev.* **D26**, 3176 (1982).
- [13] B. A. Campbell, S. Davidson and K. A. Olive, *Nucl. Phys.* **B399**, 111 (1993).
- [14] M. A. Luty, *Phys. Rev.* **D45**, 455 (1992).
- [15] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1989).
- [16] F. L. Wright *et al.*, *Astrophys. J. Lett.* **396**, L13 (1992).
- [17] Ya. B. Zel'dovich, L. B. Okun and I. Yu. Kobzarev, *Sov. Phys. JETP.* **40**, 1 (1974).
- [18] For a review see Ref. [15] above. See also P. Langacker in *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989), p 552.
- [19] J. Harvey and M. Turner, *Phys. Rev.* **D42**, 3344 (1990).
- [20] J. Ellis *et al.*, *Phys. Lett.* **B191**, 343 (1987).
- [21] L. Wolfenstein, *Phys. Rev.* **D17**, 1978 2369; S. P. Mikheev and A. Yu. Smirnov, *Yad. Fiz.* **42**, 1441 (1985); *Nuovo Cim.* **9C**, 17 (1986).
- [22] L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).
- [23] J. Donoghue, *Phys. Rev.* **D18**, 1632 (1978).
- [24] E. P. Shabalin, *Sov. J. Nucl. Phys.* **28**, 75 (1978).
- [25] E. P. Shabalin, *Sov. J. Nucl. Phys.* **31**, 864 (1980).
- [26] G. Harris and J. L. Rosner, *Phys. Rev.* **D45**, 946 (1992).
- [27] A. J. Buras and M. K. Harlander in *Heavy Flavors*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1994), p.58.
- [28] For a recent fit to the atmospheric and solar neutrino data including the full three family mixing see M. Narayan *et al.*, Institute of Mathematical Sciences preprint IMSc/95-96/001, hep-ph/9505281, May (1995).

- [29] Recent models of fermion masses almost exclusively use abelian family symmetries. See for example M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B420**, 469 (1994); P. Binetruy and P. Ramond, *Phys. Lett.* **B350**, 49 (1995).
- [30] L.-F. Li, *Phys. Rev.* **D9**, 1723 (1974).
- [31] D. Chang, R. N. Mohapatra and M. K. Parida, *Phys. Rev. Lett.* **52**, 1072 (1984); *Phys. Rev.* **D30**, 1052 (1984).